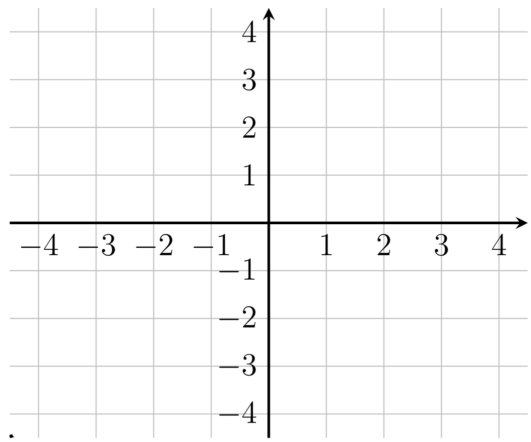
Purpose: In this problem set, you will explore transformations of functions. Using function transformations will help us take properties of the functions in our function toolkit and adjust them to answer questions about transformed functions.

1. On the coordinate axes below, sketch the triangle with vertices $(2, 4)$, $(3, 3)$, and $(0, 2)$.



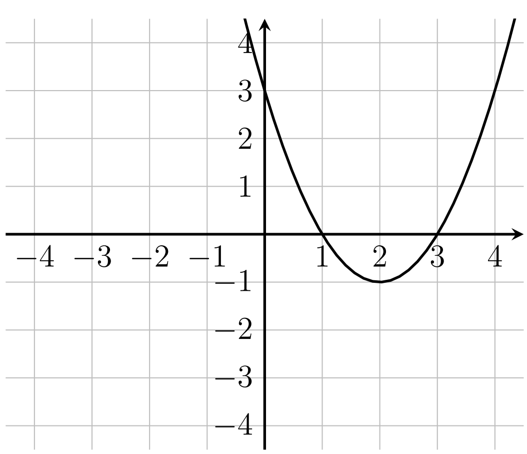
   (a) How can we shift this triangle so that the lower left vertex is at the origin?

   (b) What are the new coordinates of the vertices?

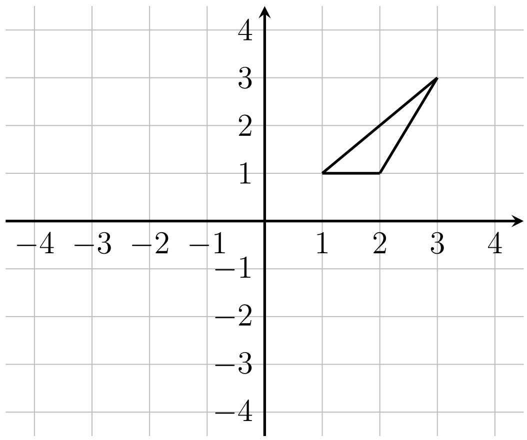
   (c) How can we shift this triangle so that the lower left vertex is at the the point $(-2, 2)$?

   (d) What are the new coordinates of the vertices?

2. How can we shift the following parabola so its vertex is at the origin?



3. What do you think the shift $(x, y) \mapsto (x-3, y+1)$ does to the graph of the triangle below?



We describe the transformation of $f(x)$ given by $(x, y) \mapsto (x+h, y+k)$ with function notation as

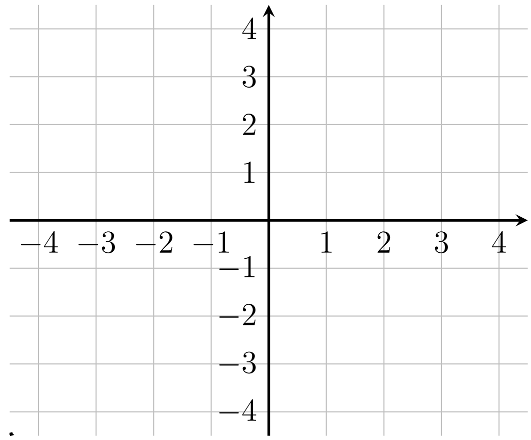
$$y - k = f(x-h) \quad \text{or} \quad y = f(x-h) + k.$$

We have two types of translations in our transformation above.

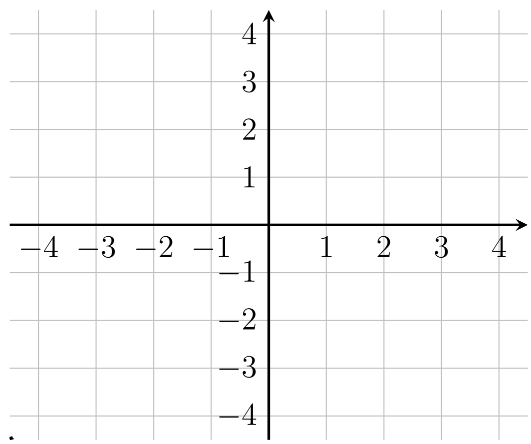
Definitions (Translations):

- Given a function $f(x)$, if we define a new function $g(x) = f(x-h)$, where h is a constant, then $g(x)$ is a _________________________ of the function $f(x)$.
  - If h is positive, then the graph will shift _________________.
  - If h is negative, then the graph will shift _________________.
- Given a function $f(x)$, if we define a new function $g(x) = f(x) - k$, where k is a constant, then $g(x)$ is a _________________________ of the function $f(x)$.
  - If k is positive, then the graph will shift _________________.
  - If k is negative, then the graph will shift _________________.

4. On the coordinate axes below, sketch the functions $f(x) = x^2$, $y = 2f(x)$, and $y = \frac{1}{2}f(x)$. How would you describe this transformation?



5. On the coordinate axes below, sketch the functions $f(x) = x^2$, $y = f(2x)$, and $y = f\left(\frac{1}{2}x\right)$. How would you describe this transformation?

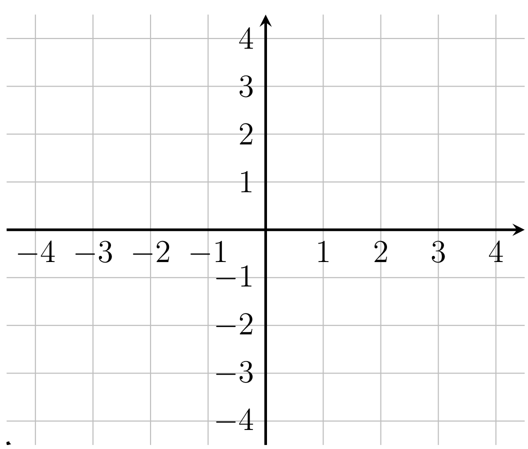


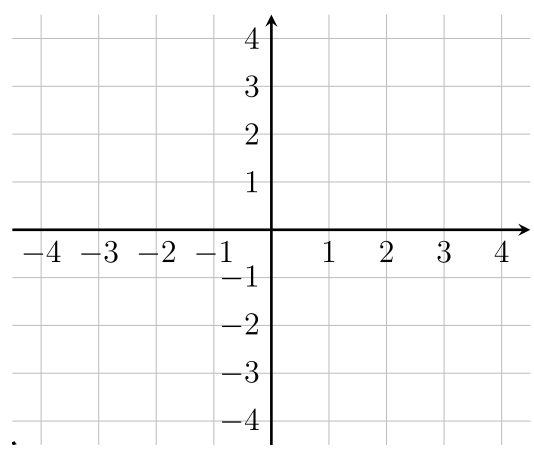
6. How might you describe the transformations $y = f(ax)$ and $y = bf(x)$? How does the size of the constants a and b change your interpretation?

Definitions (Stretches and Compressions):

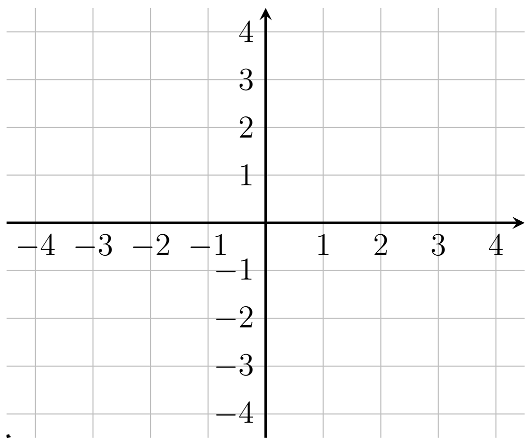
- Given a function $f(x)$, if we define a new function $g(x) = f(ax)$, where a is a positive constant, then $g(x)$ is a ______________________________ of the function $f(x)$.
  - If $a > 1$, then the graph will be ________________.
  - If $0 < a < 1$, then the graph will be ________________.
- Given a function $f(x)$, if we define a new function $g(x) = bf(x)$, where b is a positive constant, then $g(x)$ is a ______________________________ of the function $f(x)$.
  - If $b > 1$, then the graph will be ________________.
  - If $0 < b < 1$, then the graph will be ________________.

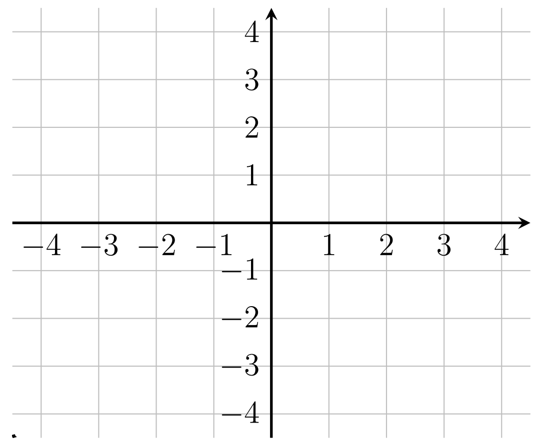
7. On the coordinate axes below, sketch the functions $f(x)$, $y = f(x)$, and $y = -f(x)$. Use one set of axes for $f(x) = |x|$ and one for $f(x) = x$. How would you describe this transformation?





8. On the coordinate axes below, sketch the functions $f(x)$, $y = f(x)$, and $y = f(-x)$. Use one set of axes for $f(x) = \sqrt{x}$ and one for $f(x) = x^3$. How would you describe this transformation?





Definitions (Reflections):

1. Given a function $f(x)$, if we define a new function $g(x)$ as $g(x) = -f(x)$, then $g(x)$ is a ________________________ of the function $f(x)$.

2. Given a function $f(x)$, if we define a new function $g(x)$ as $g(x) = f(-x)$, then $g(x)$ is a ________________________ of the function $f(x)$.

Summary: Given a function $f(x)$, write the function notation giving the transformation next to each description.

- Horizontal shift left h units:

- Horizontal shift right h units:

- Vertical shift up k units:

- Vertical shift down k units:

- Horizontal compression by a factor of a:

- Horizontal stretch by a factor of a:

- Vertical compression by a factor of b:
- Vertical stretch by a factor of b:
- Horizontal reflection:
- Vertical reflection: